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A determination of the velocity of light

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With 12 figures in the text

Introduction

Fizeau's method of determining the velocity of light is based on the fact that a light signal takes a certain time to cover a known distance. For geodetical purposes an exact knowledge of the velocity of light is of immediate interest owing to the circumstance that, the velocity being known, the inversion of Fizeau's principle will determine a distance. Here, instead of light, radar pulses may be considered just as well. In a vacuum there is no reason to assume a difference in velocity between light and radar pulses. For an accurate determination light signals, which have but little scattering and bending, are to be preferred.

In 1941 with the aid of a grant from the Längman Foundation the writer started some preliminary experiments according to Fizeau's principle at the Nobel Institute of Physics. The experiments were continued, still at the Institute, with the aid of grants from the Swedish State Council of Technical Research, and in 1947 they resulted in an apparatus for "Measurement of distances by high frequency light signalling." (18)[1] Tests were carried out at Lovö, close to Stockholm. The results were so promising that The Geographical Survey and the Gasaccumulator Co, ltd (AGA), together paid for a new technically improved and more sturdy apparatus. It was built by AGA and included a lot of the actual parts from the old one. In the autumn of 1948 a "Preliminary determination of the velocity of light" (14) was carried out by the not yet fully completed AGA-model, the "geodimeter." During the winter of 1948–49 the geodimeter was ready. The velocity measurements described below were performed in the spring and autumn 1949 at the 7 km. Geographical Survey base line near Enköping. In August measurements of distances were effected in Norrland.

General description of the apparatus and its employment

Fig. 1 and 2 shows the chief parts of the geodimeter and their use.

By means of a lens, P_1, the image of a small constant light-source, L, is projected between the plates of a Kerr cell, Kc. On each side of the cell there are the crossed Nicols prisms, N_1 and N_2. In front of the prism N_2 the intensity of the light depends on the tension between the plates in accordance with the equation:

$$J = J_0 \cdot \sin^2 k \cdot V^2 \tag{1}$$

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[1] Ref. p. 150.

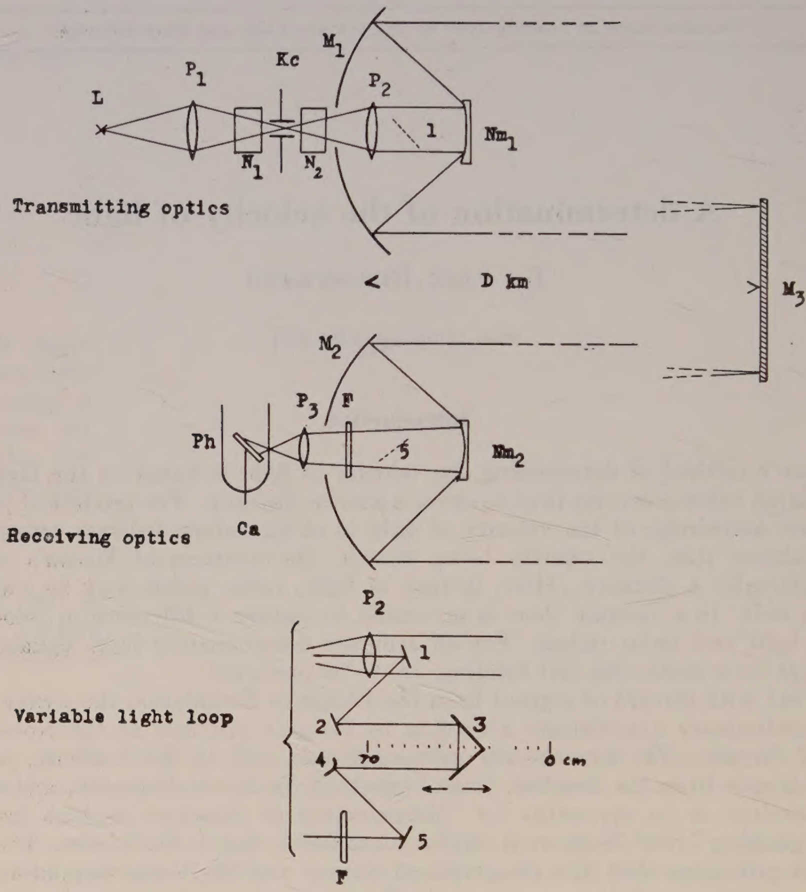

Fig. 1.

where J = intensity, J_0 and k = constants, V = difference of potential between the plates. The intensity of the light as a function of the tension is shown by Fig. 3. Between a and b the curve is almost straight. If the cell has a constant bias tension of rest between these values and an alternating tension of limited amplitude is superimposed, the intensity curve of the light is almost identical with that of the alternating tension. Added to this is a quantity of constant light depending on the bias tension. Via the tuning coils, Tc (Fig. 2), and the amplifier, HA, the plates of Kc are fed with high-frequency tension from the crystal oscillator Cr. The amplitude is 2000 volts and the frequency 8.33 Mc. Instead of the constant bias tension there is a 50-cycle alternating tension with an amplitude of 5000 volts. The 50-cycle oscillation curve being rectangular in shape (⊓⊔⊓⊔⊓⊔), it gives the most favourable conditions. The result is a completely constant tension of rest half-way between a and b, and this tension changes its sign 100 times a second. A reasonably good rectangular out line is obtained by means of a chain of neon tubes G and the resistances .05 placed over the secondary coil of the transformer Tr. Thus the tops

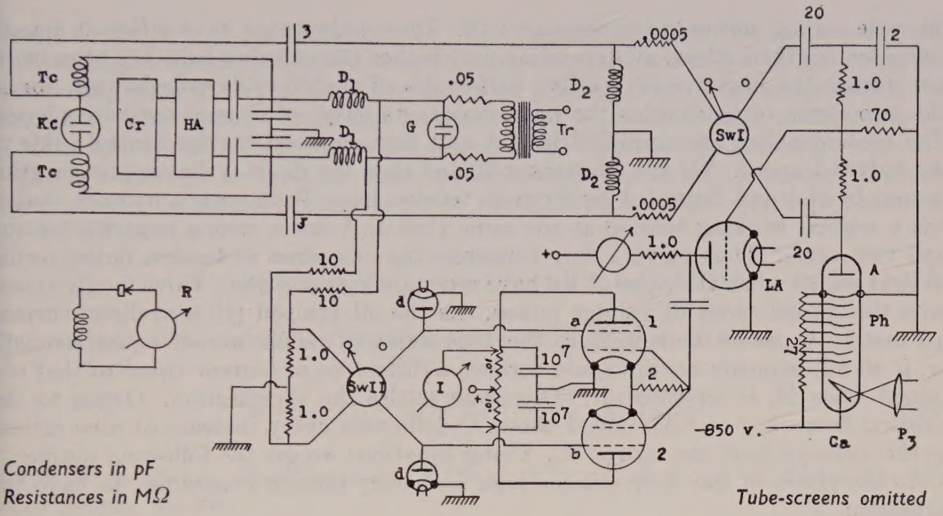

Fig. 2.

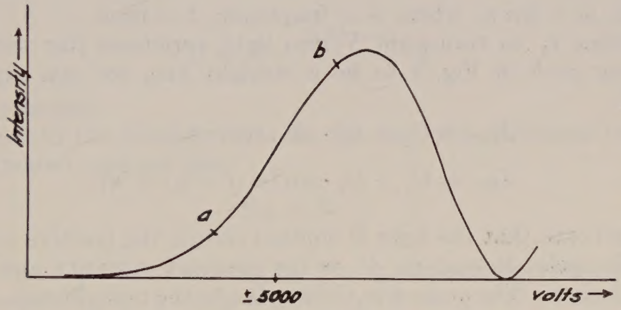

Fig. 3.

of the sine tension are cut off. The chokes D (throttles) prevent the high frequency from short-circuiting. From the size of the amplitudes and Fig. 3 it is evident that the high frequency variations change their phase simultaneously with the change of sign of the Kc-poles. This takes place 100 times a second owing to the feeding from Tr.

The transmitting optics in Fig. 1 project the image of L to the distant area where the plane mirror M_3 is set up. M_3 reflects the light straight-back and thus a small part of the image of L falls upon the receiving optics, where it is absorbed by the phototube Ph. Ph is a multiplier cell, fed with high frequency by a branch from the line to Kc (Fig. 2). The small condensers 3 and the H. F.-chokes D_2 keep the 50-cycle tension away from the phototube. The resistances 1, the condenser 2 and the anode capacity of the cell constitute a bridge preventing high frequency tension from reaching the amplifier LA. The phase of the high frequency feeding can be changed by means of the pole-switch SwI.

Owing to the rate of the feeding tension being in phase with the incoming light variations, the photocurrent will give rise to a series of more or less strong rectified

50-cycle voltage pulses in the resistance 70. These pulses may have different magnitudes among themselves, all depending on whether the entering light has been emitted during the negative or positive half-cycles of the 50-cycle polechanges, for in the two series of half-cycles the light variations have, of course, opposite phases. The tension pulses are amplified by LA and then imposed on the control grids of the valves 1 and 2. Via the resistances 10 and then the diods d the suppressor grids (a and b) of 1 and 2 are fed by 50-cycle tension from Tr in such a manner that a has a normal working tension at the same time as b has a strong negative tension and vice versa. Thus valve 1 only transmits the one series of tension pulses owing to light issued when the poles of Kc have a certain sign position. Valve 2 only transmits the second series of voltage pulses. As the oil-damped (10 sec.) direct current μA-meter, I, shows O-deflection, the two series of pulses are of equal strength or, if all adjustments are not quite correct, a difference of current equal to that obtained when M_3 is screened off. This state settles the O-deflection. Owing to the existing frequency of 8.332 Mc, I shows O-deflection every increase of nine meters in the distance from the mirror M_3. Using equations we get the following outline:

At the plates of the Kerr cell the high frequency tension is assumed to have the equation:

$$V = a \cdot \sin \omega t \tag{2}$$

a = amplitude, $\omega = 2 \pi n$, where n = frequency, t = time.

It takes a time t_1 to transform V into light variations just outside the prism N_2. Considering a—b in Fig. 3 to be a straight line, we can express the light intensity by

$$J_{50^+} = C_1 + C_2 \cdot \sin \omega (t - t_1) \tag{3 a}$$

$$J_{50^-} = C_1 + C_2 \cdot \sin [\omega (t - t_1) + \pi] \tag{3 b}$$

50^+ and 50^- indicate that the light is emitted during the positive or negative half-cycles of the 50-cycle alternation. C_1 is the constant intensity and C_2 the amplitude of the variations. The angle π in (3 b) is due to the pole change.

When it has passed the distance D to the plane mirror M_3 and back again, the light or part of it is absorbed by the phototube. During the positive *high frequency* half-cycles we get current from the anode. Assuming early saturation, we get the photo-current:

$$i_{50^+} = \tfrac{1}{2} \cdot n \cdot \int_{t_3}^{t_3 + \frac{n}{2}} [A + B \cdot \sin \omega (t - t_1 - t_2 - 2\,D/c)] \cdot dt \tag{4 a}$$

$$i_{50^-} = \tfrac{1}{2} \cdot n \cdot \int_{t_3}^{t_3 + \frac{n}{2}} [A + B \cdot \sin (\omega (t - t_1 - t_2 - 2\,D/c) + \pi)] \cdot dt \tag{4 b}$$

A and B correspond to C_1 and C_2 of eq. (3). In the time quantity $2\,D/c$ (c = velocity of light), $2\,D$ is the whole distance from the prism N_2 (or, more exactly, from the point where t_1 in eq. (3) was reckoned) to the mirror M_3 and exactly the same distance back again. The cathode of Ph is assumed to be here. t_2 is practically the running time of the photoelectrons (2.4×10^{-8} s.) from cathode to anode. t_2 may be considered to include a further small quantity of time, due to the cathode not being placed at the end of the distance $2D$. The summation in eq. (4) is done for one positive high frequent half-cycle, which begins at t_3 and ends at $t_3 + n/2$, t_3 being the transferring time of the tension from the Kerr plates to the photo anode and n the frequency. The charge per second or the current is obtained after multiplying by $n/2$, the number of half-cycles in one second of 50^+ and 50^- respectively.

The two currents i_{50^+} and i_{50^-} have opposite influences on the instrument I, which thus makes a deflection proportionate to the difference of the currents. From eq. (4) we obtain:

$$i = i_{50^+} - i_{50^-} = \frac{B}{\pi} \cdot \cos \omega \, (t_3 - t_1 - t_2 - 2\,D/c) \tag{5}$$

The current will be $i = 0$ to values on D, which fulfill the condition:

$$\omega \cdot (t_3 - t_1 - t_2 - 2\,D/c) = \frac{\pi}{2} - N \cdot \pi \tag{6}$$

where N denotes a whole number. With $\omega = 2\pi n$ and $c = \lambda \cdot n$, where $\lambda =$ the wave length, we get:

$$D = \frac{n}{2}(t_3 - t_1 - t_2) \cdot \lambda + \frac{2N - 1}{8} \cdot \lambda^1 \tag{7}$$

or

$$D = K + \frac{2N - 1}{8} \cdot \lambda \tag{8}$$

where K is a constant.

D are distances to the plane mirror. In the case of a difference between two D, K will be eliminated and we get:

$$D_N = N \cdot \frac{\lambda}{4} \tag{9}$$

D_N being known we get λ. N is easy to determine from approximate values. The frequency corresponding to λ being n and known, we get the velocity in air from:

$$c_a = n \cdot \lambda \tag{10}$$

The actual frequency of 8.332 Mc makes $\lambda = 36.0$ m.

In practice the distant mirror is placed close to a 0-point (as read off on the μA-meter). The constant crystal frequency is changed by a small (< 0.1 ‰) definable amount until 0-deflection is reached.

This single reading off is impaired by a systematic unknown error, dependent on the location of the mirror M_3 in the image of the light source. Different parts of the image have different phases, dependent on the part from which the light is emanating in the space between the Kerr plates. In Fig. 4 is shown the manner in which the change in distance corresponding to the variation of phase varies from the positive to the negative Kerr plate.

The curve is obtained at an appropriate distance D by progressively placing a very small mirror in different parts of the image of light.

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[1] Without making any restrictive assumptions about amplitudes or rectilinearness Mr. J. Clendinning (priv. com.) has proved the general validity of the eq. (7) by putting the total applied voltage in the place of V in eq. (1) and after expansing he thus obtains eq. (5) in mere terms of the form: $R \cdot \cos p \cdot \omega \, (t_3 - t_1 - t_2 - 2\,D/c)$. $R =$ num. const. $p =$ an odd integer.

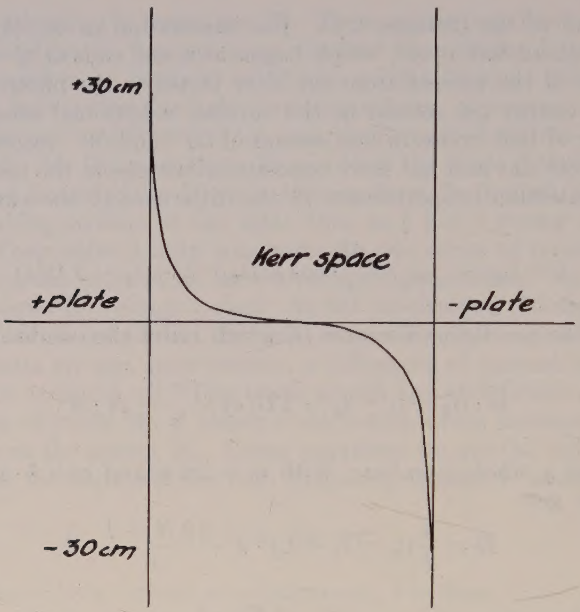


Fig. 4.

The error will not be eliminated by the change of poles, because of the simultaneously changing of phase of the valves 1 and 2 (Fig. 2). However we get two distances D if the wires to the Kerr plates are shifted, and by taking the mean we get a correct value because of the inverse symmetry of the curve in Fig. 4. In this way the error will be eliminated. In stead of a shifter at the high tension poles of the Kerr cell, two low tension shifters SwI and SwII are inserted, one in the high frequency feeding circuit to the photocell and the second in the 50-cycle circuit to the grids a and b (Fig. 2). Thereby a distance D is determined from the mean of four determinations corresponding to the four possible combined positions of the shifters SwI and SwII.

To obtain the difference of two D:s according to eq. (9) we also have to place the plane mirror at a comparatively small distance. In this case the adjustment cannot be effected by changing the frequency, as the necessary change would be too large. Instead there is a movable mirror at $D = 90$ m. The mirror being convex, the image to the geodimeter is equivalent to that from the distant mirror.

If K in eq. (8) really was a constant, K could be determined once for all. K however varies somewhat with t_1, t_2 and t_3. The variation hitherto has usually been less than two cms. Nevertheless it must be determined at each individual measurement. It is difficult to make a rapid change from $D = 90$ to $D = 7000$ m. Because of that the distance $D = 90$ is used to calibrate a small loop of light, continuously variable and including the first zero-point where $N = 1$ (eq. 8) and $D = 1$ m or so. In Fig. 1 we see the arrangement of the variable loop. The resistances .0005 in Fig. 2 effect an appropriate increase of t_2 and K with it (eq. 8). Thus the zero-point is brought to somewhere near the middle of the variable loop. Unlike the light coming from the distant mirror, that passing through the loop emanates from all parts of the light source. The two paths of light are fully comparable only if the inverse symmetry of the curve in Fig. 4 is absolute. Not relying fully on this, we graduate the loop by measurements at $D = 90$ m, and we can do that because a change of K will have the same effect in the loop as in the 90-m path. In all these measurements the value of D is a mean of four determinations as mentioned above. Such a mean did not show any marked change if the light used came from the middle or the one or the other half of the space between the Kerr plates. Thus the situation of the image of L between the Kerr plates need not be exact, and the inversed symmetry of the curve in Fig. 4 seems to be very good.

The different parts of the geodimeter will now be described a little more in detail.

Description in details

Optics

The source of light L (Fig. 1) is the incandescent spiral wire of a Luma projection lamp, T 5, 6 volts, 5 amperes, the dimensions of the spiral being 2 × 2 mm. The position of the holder is adjustable. The single positive lens P_1 is placed half-way between L and the image of L between the Kerr plates. The distance object-image is 22 cm. N_1 and N_2 are ordinary Nichol prisms. The lens P_2 is positive and its focal distance 20 cm. The plano-concave lenses, Nm_1 and Nm_2, focal distance —12 cm, are silvered on their plane backs and adjustable vertically and horizontally. The mirrors M_1 and M_2 are exactly spherical, diameter 46 cm and focal distance 75 cm. They are cut very accurately ($\pm 2''$) by Arenco, ltd. The silvering on the front is applied by vapourization ("Vacuum-Metall"). The spherical aberrations of M and Nm are the same but their signs are opposite. Only yellow light being used, the cromatic aberration is insignificant. The lens P_3 is positive and the focal distance is 2.5 cm. It projects an image some tenths of a millimeter in width in the aperture of the light protecting cope of the phototube. In front of P_3 is placed a green-yellow filter F. It is composed of one green and one yellow ordinary photographic filter.

The plane mirror M_3 is adjustable vertically and horizontally. Its diameter is 33 cm. The reflecting surface has been obtained by aluminium vapourization (AGA). For directing purposes there is a field-glass fitted with a hair cross. The curvature radius of the mirror will probably be larger than 5 kms. The convex mirror Cm of the 90-m measurements (Fig. 10) has a curvature radius of 0.5 m. The mirror is movable along a guide over a 50-cm scale S_2.

The variable loop is shown in Fig. 1. The small plane mirrors 1, 2, 5, and a convex mirror 4 (radius 6 cm) are fixed. The right angled mirror 3 is movable on a bar furnished with the 0–70 cm scale S_1. 1 and 5 can be moved aside, when the ordinary measurements are made. 4 being convex, an appropriate quantity of light reaches the photo cathode.

The Kerr cell (1) Kc (Fig. 5) has circular electrodes of rustless steel. They are 8 mm in diameter, and the distances between them is 2 mm. The onepiece glass vessel has been made and the electrodes fixed by Freiberger & Andrae. The liquid is nitrobenzene purified by shaking for three hours with an equal weight of aluminium oxide, threefold destillation in vacuo (retort and receiver in one piece), the first and last thirds of the distillate being rejected each time, and finally by high tension electrolysis for one hour. In order to remove absorbed air the nitrobenzene was shaked in the vacuum before each distillation. Special precautions were made to keep air out of the nitrobenzene during the whole purifying process. After being filled the Kerr cell was sealed by fusing.

With a tension of 5000 volts 50-cycle plus 2000 volts high frequency (8 Mc) the temperature of the cell will rise to some 60° C above that of the surroundings. A small fan reduces this additional temperature to 20° C. After a 300 hours' run there was no observable change in the cell.

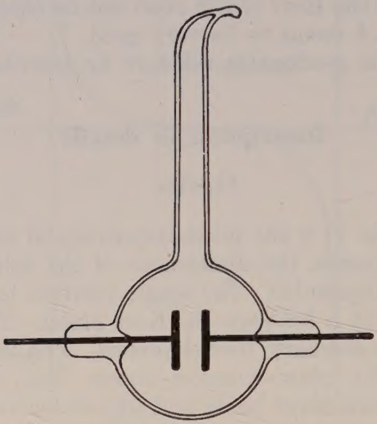


Fig. 5.

The effective wave length of the light used depends mainly on the filter, the transmission of nitrobenzene and the sensitivity of the phototube. The following determination was performed: A slit was placed outside the Kerr cell; with a minimum of deviation, the light was passed through a crown glass prism and a projecting lens. From the resultant spectrum different parts were selected by means of a narrow slit. After passing the filter, F, the selected light was absorbed by the photocell, and the photocurrent was measured. With 5 volts on the lamp, L, normal running tensions in the Kerr cell, the sensitivity curve shown in Fig. 6 was obtained. The curve is corrected with regard to the variation in dispersion with the wave length. The colour was judged by the naked eye. As the successive movements of the slit over the spectrum were always made in equal stages, the adapted and adjusted colour-scale cannot on the whole be displaced by more than 50 Å. When we consider that the index of the refraction of light indicates the change in the velocity in vacuum as compared to that in air, and that the mentioned index includes a term B/λ^2 ($\lambda =$ wave length) we get from the curve an effective index of refraction corresponding to the wave length 5440 Å at 5 volts on the lamp. Knowing the ordinary temperature of 2540° K at 6 volts (from the manufacturer) we get the black temperature at this and other voltages from the usual tables. Thus the lamp voltage was usually 2.8–3.5 volts during the measurements. According to a table by Pirani this means 1900°–2130° K or 1770°–1960° K black temp. By Wien's emission formula the curve shown in Fig. 6 can be transformed to the temperature of 1770° K black temperature corresponding to 2.8 volts on the filament. The effective wave length then has changed to 5570 Å. In view of the blue absorption of the air and of the fact that this absorption was proportionally larger when higher filament tensions than 2.8 volts were used a refraction index corresponding to 5600 Å was used throughout.

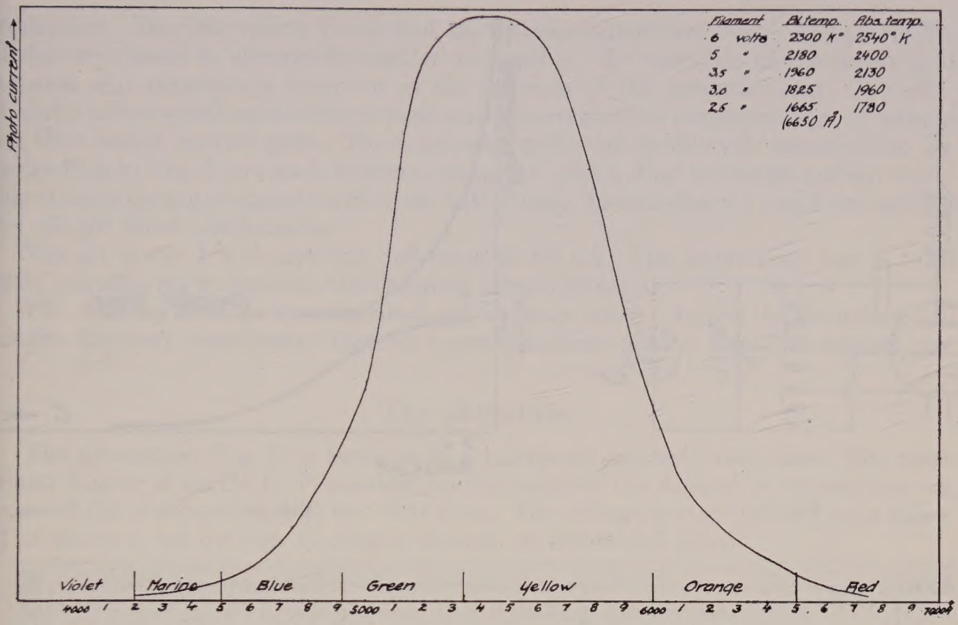


Fig. 6.

The crystal oscillator with amplifier

The circuit of the crystal oscillator is shown in Fig. 7. By the rotating condenser C the frequency can be varied. Cr is the crystal. Fig. 8 shows the frequency as a function of the capacity of C. The crystal itself is of an older type held between plane brass electrodes. The protecting box of aluminium contains a contact thermometer and a heating wire. The whole set thus forms a thermostate. The frequency continuously increases by itself during running conditions. The increase seems to be of the expected magnitude. Every measuring night the frequency was checked by wireless transmission to the check station of The Telegraph Service.

The capacity C having large values, the crystal cools down owing to looser coupling. The consequence is a fall in frequency. Therefore C was kept within reasonable limits. The first hour after the start the increase in the frequency is considerable, and thus the measurements during this period are uncertain.

In table I are shown all checks of the frequency for the different values on the scale of C. A working curve (Fig. 9) of the frequency depending on the C scale has been plotted from table I, after a parallel reduction of all values to 8332230 Hz ("base frequency") at C = 4.7. No objection to such a reduction can be detected from the table nor is it to be expected within the limited frequency interval of 400 cycles, where the measurements were made. The capacity to the circuit is thereby changed from 50 to 65 pF.

After the quartz oscillator there is the H. F.-amplifier HA (Fig. 2) in three steps, the two last of which have pushpull arranged double valves. As is to be seen in Fig. 2 the outgoing wires from HA thus come directly from the choked anodes of the power valve, the h. f. power of which is some 40 watts.

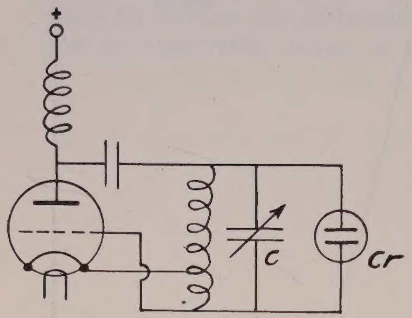


Fig. 7.

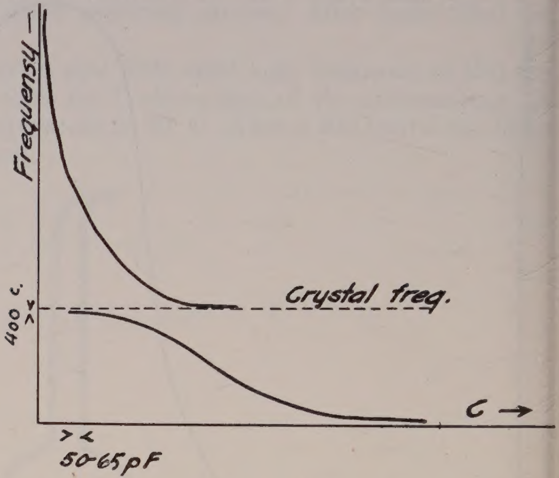


Fig. 8.

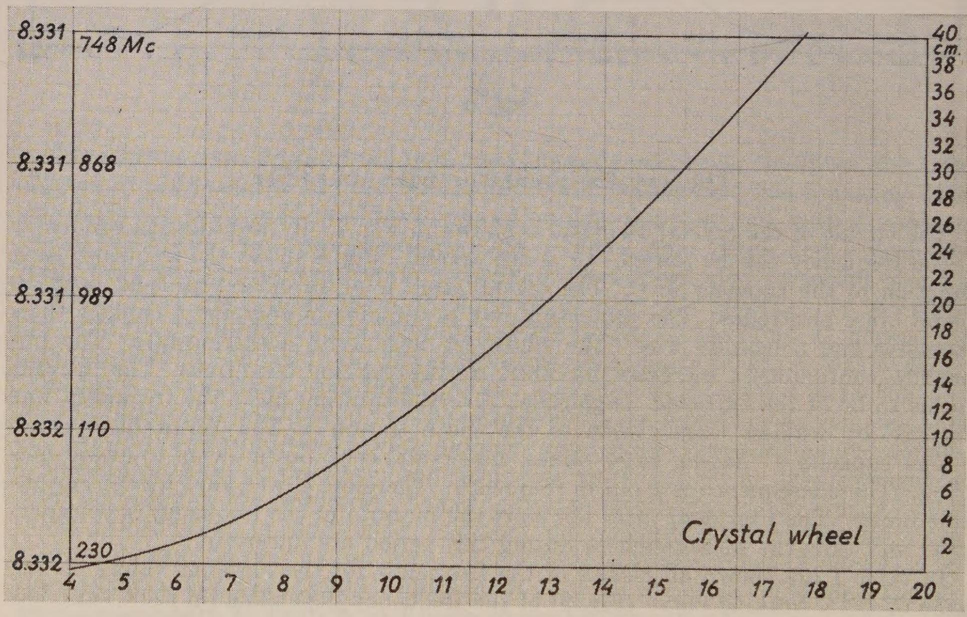


Fig. 9.

The balance circuit

The 70 MΩ resistance of the photoanode is at the same time the grid resistance of a low frequency amplifying valve LA (Fig. 2) in front of the specific balance circuit. To compensate for the voltage drop due to the direct component of the photo current, there is a possibility of varying the tension at the earthed end of the resistance. The two valves 1 and 2 of the balance circuit are steep, Philips EF 50. They are chosen as electrically similar as possible. By variation of the screen-grid tension and appropriate positions of the contacts of the potentiometer, .001-.001, 1 and 2 effect equal variations in their anode currents to a certain change of tension on their joined control grids. The suppressor grids, fed by 50-cycle tension from Tr, according to Fig. 2, are each directly connected with a diod to ensure normal working tension during the positive 50-cycle half-cycles. The anodes of 1 and 2 are earthed by 10 μF block condensers.

The μA-meter I will give full deflection to 60 μA. The instrument box is filled with paraffin oil to increase the damping effect (10 sec).

The running tensions are smoothed out by neon tubes. Across the secondary coil of the filament transformer there is a potentiometer with a movable earthed tap.

The phototube

The phototube (Fig. 2) is enclosed in a lightproof earthed brass cope. The ninth (last) binode is earthed. Proceeding to the cathode the tension is successively increased (by a potentiometer) to —850 volts. The voltage is stabilized by neon tubes. The anode is fed by high frequency tension, as mentioned above.

Dependence on the running conditions

The distances to the successive zero-points (eq. 8) are increased by 5 mm per one per cent of the increase in the supply voltage (more rapid photo-electrons). These displacements are the most common among those which have to be compensated for by the readings on S_1. The h.f.-amplitude, kept under manual control, has a slight effect on the zero-points too. One per cent increase in amplitude shortens the distances to the zero-points by 0.6 mm.

A change of 500 % in the intensity of the received light has no detectable influence on the positions of the zero-points.

Instruments and controls

1) The zero instrument, I, has already been mentioned in "The balance circuit."
2) Voltmeter to the supply voltage.
3) Voltmeter to the lamp L.
4) Wheel to the rheostat of L.
5) Wheel of a turnable copper disk in the tuning coil of the Kerr cell to compensate the resonance change caused by the capacity decrease of the warmed cell. The dielectrical constant of the nitrobenzene decreases.
6) Wheel of a rheostat in the screen circuit of the H. F. power valve, whereby the H. F.-amplitude is kept constant when fluctuations in the supply tension are considerable.
7) M. A.-meter of a small crystal receiver R (Fig. 2), showing the H. F.-amplitude and resonance of the Kerr cell circuit.
8) Frequency wheel of the crystal oscillator.
9) M. A.-meter in the anode circuit of the photo current amplifier LA, showing the magnitude of the direct current component of the photo current, including dark current and correct bias tension of LA.

10) Zero instrument in series with I for rough adjustments.
11) Movable taps from the anode resistances of the two balance valves and leading to the zero instruments.
12) Variable screen-grid balance of these valves.
13) Variable earth point of the balance circuit filament current.
14) Variable bias tension of LA.
15) Phase shifter SwI of the photocell H. F.-feed.
16) Phase shifter SwII of the 50-cycle feed of the suppressor grids of the balance valves.
17) Movable mirror 3 (Fig. 1) with scale for reading off the position of the first zero-point.
18) Thermometer for reading off the temperature in the crystal oscillator box.
19) Thermometer for the air temperature along the path of light out to the plane mirror.
20) Barometer for the air pressure along the same path.

Readings off

Measuring against the distant mirror, we read off the frequency wheel after each change in the phase shifters, the deflection on instrument I being zero. Zero deflection is equal to that when the plane mirror is screened off. This deflection of course by adjusting the balance circuit, is to be kept close to that caused by zero current through instrument I. After each group of four readings of the frequency wheel, corresponding to the four possible combinations of the positions of the phase shifters, analogous readings S_1 are to be made of the position of the movable mirror at zero deflection of the instrument I. Measuring against the convex mirror at 90 m will be described below.

Among the remaining instruments that for the H. F.-amplitude is to be kept under constant supervision. The wheels 11), 12) and 13), as above, are adjusted once for all. The remaining wheels are usually adjusted before each measurement series. Air pressure and temperature are read off once an hour. These observations are necessary because the velocity is dependent on the atmospherical conditions. This will be dealt with in next section.

Influence of the atmosphere

Refractive index of dry air

With the present measurements, the changes in the refractive index of the air with changes in temperature, pressure and humidity play an important rôle. To transform the obtained value on the velocity to corresponding value for vacuum we also have to know the real magnitude of the refractive index.

The increase in the velocity of light on transition from air to vacuum is supposed to be proportional to the refractive index. This is applicable to the wave velocity. If the refractive index of the air is expressed by

$$\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} \tag{11}$$

where A, B, C, are constants and λ the wave length, we get, after taking the present case with *group velocity* (3, 9, 14) into account, a corresponding "refractive index for group velocity" suited to the computations:

$$\mu_g = A + \frac{3\,B}{\lambda^2} + \frac{5\,C}{\lambda^4} \tag{12}$$

For dry air at 0° C, 760 mm Hg, Barrel and Sears (2) gives the following expression of the wave refraction index:

$$\mu_0 = 1 + \left(2876.4 + \frac{16.288}{\lambda^2} + \frac{0.136}{\lambda^4}\right) \cdot 10^{-7} \tag{13}$$

with λ in thousands of a millimeter.

After transforming to group velocity, we get, with $\lambda = 0.0005600$ mm or 5600 Å:

$$\mu_B = 1.00030388 \tag{14}$$

Using the formula of Perard (Trav. Bur. Int. Poids Mes. 1934) we get:

$$\mu_P = 1.00030382 \tag{15}$$

According to Köster and Lampe (Phys. Zeit. 1934):

$$\mu_0 = 1 + \left(2877.57 + \frac{15.843}{\lambda^2} + \frac{0.193}{\lambda^4}\right) \cdot 10^{-7} \tag{16}$$

we get

$$\mu_K = 1.00030390 \tag{17}$$

Bureau of Standards' value (1918) used in 1948 (14) is obviously too low. The above figures agree with the mean of 40 values after 1857 (2) and with recent values obtained by the determination of the yard and meter in wave lengths (2). Thus for 5600 Å the following value is used:

$$\mu_g = 1.0003039 \pm 0.0000002 \tag{18}$$

The variations of μ_g with λ, t, p, e

From eq. (12) we get the variation with the wave length as

$$\frac{d\,\mu_g}{d\,\lambda} = -\,0.00000055 \ \text{per } 100 \text{ Å} \tag{19}$$

If the temperature is t° C, the pressure p mm Hg and the humidity e mm Hg, the refractive index of air, according to Kohlrausch, will be:

$$\mu_L = 1 + \frac{\mu_0 - 1}{1 + \alpha t} \cdot \frac{p}{760} - \frac{0.00000055\, e}{1 + \alpha t} \tag{20}$$

where $\alpha = 0.00367$ ($\simeq 1/273$).

After replacing μ_0 by μ_g we get the variations of μ_g with temperature, pressure and humidity at 760 mm Hg and

$$0^\circ\ \mathrm{C}:\quad \frac{d\,\mu_g}{d\,t} = -\,1.11 \times 10^{-6} \tag{21}$$

$$+\,10^\circ\ \mathrm{C}:\quad \frac{d\,\mu_g}{d\,t} = -\,1.04 \times 10^{-6} \tag{22}$$

$$0^\circ\ \mathrm{C}:\quad \frac{d\,\mu_g}{d\,p} = +\,0.398 \times 10^{-6} \tag{23}$$

$$+\,10^\circ\ \mathrm{C}:\quad \frac{d\,\mu_g}{d\,p} = +\,0.384 \times 10^{-6} \tag{24}$$

$$0^\circ\ \mathrm{C}:\quad \frac{d\,\mu_g}{d\,e} = -\,0.055 \times 10^{-6} \tag{25}$$

$$10^\circ\ \mathrm{C}:\quad \frac{d\,\mu_g}{d\,e} = -\,0.053 \times 10^{-6} \tag{26}$$

The variation of all these quantities with variations in pressure may be neglected here as being less than 10^{-9} pro mm Hg.

Determination of the atmospheric conditions

To determine the air pressure, a Hg-barometer was placed on a level with the light path. The barometer was checked by the Meteorological Service to be correct to within 0.05 mm.

The temperature was read off by means of thermometers at both ends of the measured distance. They were placed 0 and 2 m above the light path and 4 and 3 m above the ground, which at both ends had an inclination of 1 : 10 down to the fields, situated 15 m beneath the light path. The thermometers were graduated in tenths of degrees C and checked to be correct to within $\pm\,0^\circ\!.03$ C.

As regards the temperature gradient above the ground, Johnson and Heywood (20) have values for every month of the year and every hour of the day, obtained above ground similar to that in the present case. Allowing for as light bend of the isotherms along the ground, the one for the light path has been supposed to pass 4 m above the site of the termometer place. The temperature gradient was estimated with Johnson and Heywood's curves, to be some $+0^\circ\!.05$ C per meter, thus giving an average correction of $+0^\circ\!.2$ C in the temperatures read off.

The humidity was estimated to be 70 % in the spring nights and 85 % in the autumn nights.

Measurements on the field

Adjustments

The image of the lamp filament is arranged to fall between the Kerr plates. The optical axis of the mirrors and lenses are arranged to coincide with each other. When the back Nicol has been temporarily replaced by a small inclined mirror, the image of some distant object, the top of a tree or the like, is to be seen in the mirror between the Kerr plates. By moving the mirror Nm_1 (Fig. 1), the optical system is focused until the image remains clear of parallax. The searcher field glass of the unmoved geodimeter is then adjusted until it is directed towards the same object. By use of the searcher and with unchanged transmitting optical system we then can direct the geodimeter, i.e. the beam of light towards the distant mirror. At this end of the distance the cooperator systematically sweeps the perpendicular of the mirror plane over the geodimeter region by small turnings of the adjusting screws of the mirror. After a few minutes the reflected light is to be seen as a glimpse at the geodimeter end and with the help of agreed light signals the mirror soon will be adjusted towards the receiving optical system. By means of a small inclined auxiliary mirror in front of the filter F the receiving optical system is adjusted so that the focused beam falls upon the aperture of the lens P_3 (Fig. 1). In the case of a strongly reflected light or a specific signal lamp, the beam will be seen as a projected spot. This facilitate the adjustments.

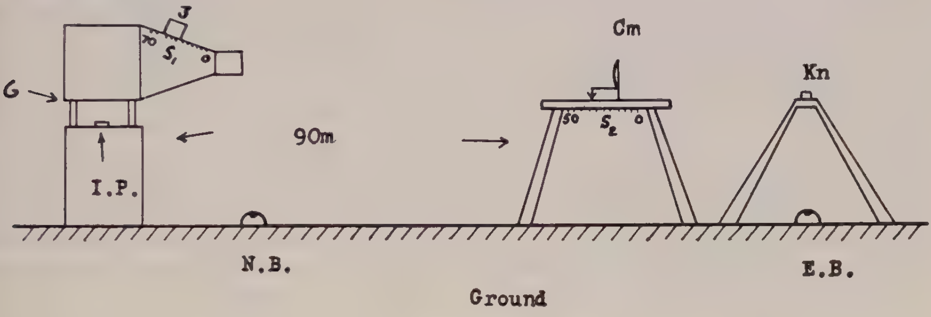


Fig. 10.

The position of the first zero point

To determine the position of the first zero-point in relation to the geodimeter index, measurements were performed over a distance of 90 m. In Fig. 10 we see the arrangement at the northern end of the Enköping base line.

G is the reference index, placed in the middle of the bottom back edge of the geodimeter. I. P. is the index knob of the concrete instrument pillar. 3 is the movable mirror of the variable loop having the scale S_1 0–70 cm. N. B. is the northern ground knob of the 7 km base-line. Cm is the small movable convex mirror on its foot with the scale S_2 0–50 cm. E. B. is the ground knob of the small "evaluating base" at a distance of 90 m from I. P. Kn is an index knob, plumbed from E. B. G, I. P., N. B., Cm and E. B. are all in the vertical plane of the 7 km base line or its extension. I. P., S_2 and Kn are on a common straight line.

The specific measurements were made in the following way:

On the scale S_1 was read off the position of mirror 3 at 0-deflection on the instrument L. Four readings were obtained corresponding to each of the four possible position combinations of the shifters Sw I and Sw II. The mean of the four readings was the value taken. With the image of the lamp filament projected towards the mirror Cm, a corresponding series of readings of its positions were made on the scale S_2, and the mean was taken. Knowing the distance Cm–G reduced by $10/4 \cdot \lambda$ (eq. 9), we get the distance between the first zero-point and G for a certain position of mirror 3 at I-zero reading. All readings S_1 and S_2 are assembled chronologically in table II (p. 028). S_1 are in italics. The following table gives the distance values for the 90-m determination in May:

I. P.–E. B. horizontally by invar tapes and strings	+91.0885 m	
Corr. of strings +0.40 mm per 24 m	+ 0.0012	
Corr. of tapes 1.495 mm per 4 m	+ 0.0075	
Sum	+91.0972	±0.0005 m

An altitude difference of 11.709 m causes the used distance to be

I. P.–E. B.	+91.8466 m	
E. B.–Kn horizontally (by theodolite)	+ 0.0009	
Kn–S_2 (zero line of S_2)	— 1.6470	
Cm index–Cm surface	+ 0.1220	
I. P.–G	+ 0.1320	
Corr. of optics focused to ∞	+ 0.0120	
S_1–S_2 as a mean of 13 readings (tab. II)	— 0.0526	
10/4 · λ at 8° C, 760 mm Hg, 8332240 Hz	—89.9232	
The distance G–1st zero-point Sum	+ 0.4907	±0.0021 m

With this position of the first zero-point we should obtain zero-deflection on the μA-meter, I, at the movable mirror 3 directed on the zero line of S_1.

A direct measurement of the light path in the loop at mirror position 0 also gave 0.49 m. This determination is none too certain, but corroborates the value above and shows that the curve of Fig. 4 has good inverse symmetry.

In autumn the corresponding figures were:

I. P.–E. B.	+91.8466	
E. B.–Kn	+ 0.0040	
Kn–S_2	— 1.6512	
Cm index–Cm surface (new setting)	+ 0.1235	
I. P.–G	+ 0.1285	
Corr. from focus	+ 0.0120	
S_1–S_2 as a mean of 12 readings (tab. II)	— 0.0600	
10/4 · λ at 3° C, 765 mm Hg, 8332180 Hz	—89.9230	
The distance G–1st zero-point Sum	+ 0.4804	±0.0021 m

The difference of 1 cm as compared with the spring value may very well be due to changed dimensions of the light path within the geodimeter. They were however unaltered during the respective measuring periods. Unfortunately the light path in the loop was not measured again in the autumn.

The position of the 769th zero-point

In the course of the long-distance measurements, the position of the 769th zero-point in relation to the index G is the same as the position of the plane mirror relative to G. That is so because we adjust the frequency until the zero-point and the mirror coincide. The main part of the distance from G to the plane mirror is the fixed distance from I. P. to the southern ground knob of the base line. The determination of this distance is shown in the following table:

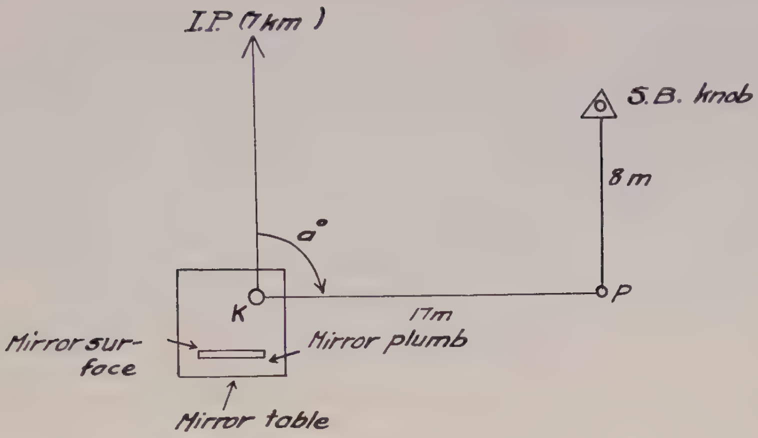


Fig. 11.

N. B. knob–S. B. knob on the ellipsoide according to the manuscript G 4102 (of the Survey)		+6885.0672	
Considering the measurements of the Baltic Geodetic Commission		+ 0.0028	
Considering safe (spare) markings		— 0.0020	
Corr. for the actual mean altitude of 27.4 m. a. s. (G and mirror)		+ 0.0296	
Corr. for the diff. in alt. betw. G and mirr. = 1.1 m		+ 0.0002	
N. B.–I. P. by invar tapes		+13.2441	
Corr. of tapes = 1.495 mm per 4 m		+ 0.0049	
Used distance I. P.–S. B. knob	Sum	+6898.3468	±0.0030 m

Then we have the adjoining distances between the S. B. knob and the mirror and between I. P. and G.

The determination of the distance between the S. B. knob and the mirror is shown in Fig. 11. 8 m backwards in the extension of the base line a pile was driven into the ground. The pile had an index knob P. At right-angles 17 m to the west there was placed the stand of the mirror, 3.5 m in height and furnished with a table and index knob K. The horizontal distance P–K was 16.92 m. The angle α was measured with a theodolite. The accuracy corresponded to 0.1 mm in the direction of the base line. Supposing the distance I. P.–P was exactly equal to I. P.–K, α would be $99^c\,92'\,20''$. α as measured was $100^c\,04'\,70''$ in May and $100^c\,05'\,52''$ in September. The differences from $99^c\,92'\,20''$ give a 3.32 and 3.54 cm respectively shorter distance I. P.–K than I. P.–P.

In a rough calculation in May, the quantity 3.32 cm was used with a wrong sign. The value of the velocity obtained 299 795.6 (this was possibly published somewhere) agreed, with that of 1948, which was very misleading.

The distances between the S. B. knob and the mirror are entered in the two following tables, the first one for May, the second for September:

S. B. knob–plumbed index	—0.0019
S. B. index–P by invar tapes	+7.9753
Corr. of invar tapes	+0.0030
Distance from the angel α (Fig. 11)	—0.0332

K–mirror plumb	+0.1648	
Plumb–mirror surface	—0.0225	
Total distance S. B. knob–mirror in May	+8.0855	±0.0005 m

S. B. knob–plumbed index	+0.0006	
S. B. index–P by steel tape	+7.9755	
Corr. of steel tape	+0.0015	
Angle α	—0.0351	
K–mirror plumb	+0.1630	
Plumb-mirror surface	—0.0225	
Total distance S. B. knob–mirror in Sept.	+8.0830	±0.0005 m

In the following table are put together the total distances between G and the mirror

Date	May 6—11	May 13—14	Sept. 24—29	Oct. 3
G–I. P.	+0.1426	.1245	.1208	.1216
I. P.–S. B. knob (see above)	+6898.3468	.3468	.3468	.3468
S. B. knob–mirror (see above)	+8.0855	.0855	.0830	.0830
G–mirror	6906.5749	.5568	.5506	.5514 m

Reduced distance from 0-point to 0-point

The adjustments of the position of the 769th zero-point on the plane mirror by means of the freq. wheel are effected for different values of the frequency (increase of fr. p. 9, 23). Furthermore, these adjustments were made under varying atmospheric conditions. To make the measurements comparable we make a reduction to 0° C, 760 mm Hg dry air and the base frequency 8332230 Hz with wheel set at 4.7 (Fig. 9). After the reductions, owing to the uncertainty of the determinations, the 769th zero-point will lie at a somewhat varying distance from the plane mirror.

When calculating such a distance we start with the readings on the frequency wheel. All these long-distance readings are assembled chronologically in the table III (p. 147–148). The groups of four (or eight, etc.) readings are taken directly from the frequency wheel. Among these groups we see the S_1-readings (in italics) of the movable loop mirror 3. The hours of start and conclusion for each night are inserted too. The thermometer and barometer readings are assembled in table IV (p. 31).

From the curve of Fig. 9 we get the distance of the 769th 0-point from the plane mirror with the frequency wheel set at 4.7. As an example we take the first readings in table III. The figure 11.6 corresponds to a difference in frequency of 185 Hz as compared with that obtained with the wheel set at 4.7. Since the total distance from G to the plane mirror is 6906 m, with 185 Hz we get a displacement of the 769th 0-point of

$$-\frac{185\times 6906}{8332000} = -\,0.153 \text{ m} \quad \text{or} \quad -15.3 \text{ cm}$$

The minus sign indicates that the point lies on the hither side of the mirror. In this way the curve in Fig. 9 also can be graduated in such cm-distances. This is done in the scale along the right margin. The three following readings on the wheel were made with the three other combinations of the shifters Sw I and Sw II (Fig. 2). As a mean of the first group of four readings we get the distance 13.9 cm. However we need the actual position of the first zero-point as determined by the position of the movable mirror 3 (Fig. 1, 10). Here, too, we have four readings (S_1) in a group. Usually we take the mean of the groups before and after the group of wheel readings. As the before going group is missed in the start we only take the succeeding one and get an S_1 of 16.5 cm. After a reduction with this quantity the whole series of zero-points will start (= 1st point) at a distance of 49.07 cm from G (from p. 134). Inserting temperature, pressure and 70 % humidity (tab. IV) in the formulae (p. 132) we get the corrections —6.1 cm, —2,6 cm and —0.2 cm.

Thus we get

Distance as from the curve in Fig. 9	—13.9
Movable mirror (S_1)	+16.5
Temp. corr.	— 6.1
Corr. of temp. corr. (p. 132)	— 0.1
Pressure corr.	— 2.6
Humidity corr.	— 0.2
Sum	— 6.4 cm

The mean of all determinations on the 6th May is —4.4 ± 0.63 cm. On an average the frequency can be considered to be 5 Hz above the values of the curve in Fig. 9. This means a further corr. of +0.4 cm. As result on the first night we get the 769th zero-point at 0° C, 760 mm Hg dry air and 8332230 Hz to be —4.0 cm on this side of the plane mirror if the 1st zero-point had caused an S_1-reading = 0. Thus the total reduced distance between the 1st and the 769th zero-point will be:

First zero-point–G (p. 132)	—0.4907	
G–mirror (p. 136)	+6906.5749	
Mirror–769th zero-point (above)	—0.0400	
1st zero-point to 769th zero-point	+6906.0442	m

Calculation of the velocity of light

The above value of 6906.0442 m is the distance D_N in eq. (9), N being 768. From the eq. (10) with $n = 8332230$ Hz, we find the velocity in dry air at 0° C, 760 mm Hg to be

$$c_a = 299701.8 \text{ km/sec}$$

The effective wave length of the light used being 5600 Å, we get, according to eq. (18) p. 131 the relation to the velocity in vacuo to be

$$\mu_g = 1.0003039$$

Thus the final result from the first night is

$$c = 299\,792.9 \text{ km/sec.}$$

In the same way we can compute the rest of the table on the other page. To begin with a computation has been made of all reduced distances from the

769th zero-point to the plane mirror. These distances have furthermore been corrected to hold for the first G-mirror position May 6–11 having the distance from first 0-point to plane mirror equal to 6906.5749—0.4907 = 6906.0842 m. From the mean error for each night has been obtained a certain weight, valid for the value obtained for that night, the starting-point being the inverse square of the night's mean error. In order that the measurements for each separate night, might have their full individual effect, the greatest weights were reduced to about half and the lowest weights somewhat increased.

In the following table the obtained cm-distances are assembled, with their weights and the number of determinations. Thus to get the zero-distances D_N (eq. 9) we have to reduce 6906.0842 by the different cm-distances, and the velocity is found from eq. (10) and (18). To illustrate the step-wise increase of the frequency with working time the means of the frequency corrections used each night are also inserted in the table. There is also a column giving the running voltage of the lamp. Owing to variable visibility, the intensity of the return light was the same all the time:

Date in 1949	Lamp Voltage	Fr. corr. cm	Number obs.	Weight 1 obs = 1.0	Dist. cm	Velocity 299 790+	Temp. C°	Pressure mm Hg
May 6	2.8	−0.4	4	2.0	4.0	2.87	+ 8	750
8	2.8	−0.4	6	12.0	2.4	3.56	+ 1	755
9	2.8	−0.6	1	1.0	4.9	2.50	+ 3	762
10	2.8	−0.7	7	5.0	4.5	2.63	+ 9	765
11	2.8	−2.5	5	9.0	4.8	2.52	+ 8	769
13	2.8	−2.5	5	3.0	2.5	3.52	+11	765
14	2.8	−2.4	6	2.0	3.8	2.93	+ 9	763
Sept. 24	5.0	−3.3	2	0.5	4.1	2.84	+11	768
26	5.0	−4.5	5	2.0	0.7	4.27	+15	764
27	5.0	−4.5	5	5.5	4.4	2.68	+12	763
28	6.0	−4.5	2	0.3	1.3	4.05	+11	764
29	3.5	−5.2	5	7.5	4.1	2.84	+14	758
Oct. 3	3.5	−5.3	5	8.2	3.0	3.32	+ 9	749

During the whole spring visibility was good. In the autumn, however, only on the two last days. No systematic influence of the haze was observed. Probably a deepening red was compensated for by the higher temperature of the lamp.

From the table we get

Mean of spring values	299 793.04 ± 0.19 km/sec.
Mean of autumn values	299 793.10 ± 0.20 km/sec.
Total mean	299 793.1 ± 0.14 km/sec.

$$\left(\pm\sqrt{\frac{[p\,v\,v]}{[p]\cdot(13-1)}}\right)$$

The limits are obtained from the deviations from the mean. To get the real limits we have to consider the uncertainty of several external conditions.

The error in the colour estimation has been set at 50 Å. According to eq. (19) this causes a relative error of $\pm 2.8 \times 10^{-7}$.

The mean error for the temperature will be less than ±0°2 C, that of the pressure less than ±0.2 mm and that of the humidity less than ±1 mm. Thus from the eq. 21–26 we get an atmospheric error of $\pm 2.5 \times 10^{-7}$.

The Enköping base line was remeasured by the Baltic Geodetic Commission. The resulting mean error of 4×10^{-7} will be increased to 5×10^{-7} by including errors in the small actual added distances.

Regarding the frequency, the mean error will not be larger than 2 Hz or $\pm 2.4 \times 10^{-7}$.

Finally the position of the first zero-point contributes with an error of $\pm 3.0 \times 10^{-7}$.

Thus totally we get the errors

769th zero-point	$\pm 4.5 \times 10^{-7}$
1st zero-point	$\pm 3.0 \times 10^{-7}$
Refr. index	$\pm 2.0 \times 10^{-7}$
Colour	$\pm 2.8 \times 10^{-7}$
Atmosphere	$\pm 2.5 \times 10^{-7}$
Base line	$\pm 5.0 \times 10^{-7}$
Frequency	$\pm 2.4 \times 10^{-7}$
$\sqrt{[vv]} =$	$\pm 8.80 \times 10^{-7}$

As final result of the determinations on the 7 kms base line we arrive at a velocity of

$$c = 299\ 793.1 \pm 0.26 \text{ km/sec.}$$

The limits include all known contributory errors.

Values from other base lines

1) As the base line above was bisected (angle $198^{c}84^{`}86^{``}$) there were two further distances which could be used for determinations. The longer one, with the instrument still at N. B., and having 573 0-points, was composed of the following distances.

1st zero-point–G	—0.4804	
G–I. P.	+0.1227	
I. P.–M. B. (along light path)	+5147.7036	±0.010
M. B.–mirror	—3.7576	
Mirror–573th zero-point (reduced)	—0.0045	
1st to 573th zero-point	5143.5838	±0.013

This gives the value of c as

$$c = 299\ 794.0 \pm 0.75 \text{ km/sec.}$$

2) Using the shortest base line, with the geodimeter at M. B., the distance components were:

1st zero-point–G	—0.4804	
G–M. B.	+4.1550	
M. B.–S. B.	+1750.9060	±0.0070
S. B.–mirror	+7.9175	
Mirror–197th zero-point (reduced)	—0.0214	
1st to 197th zero-point	1762.4767	±0.0090 m

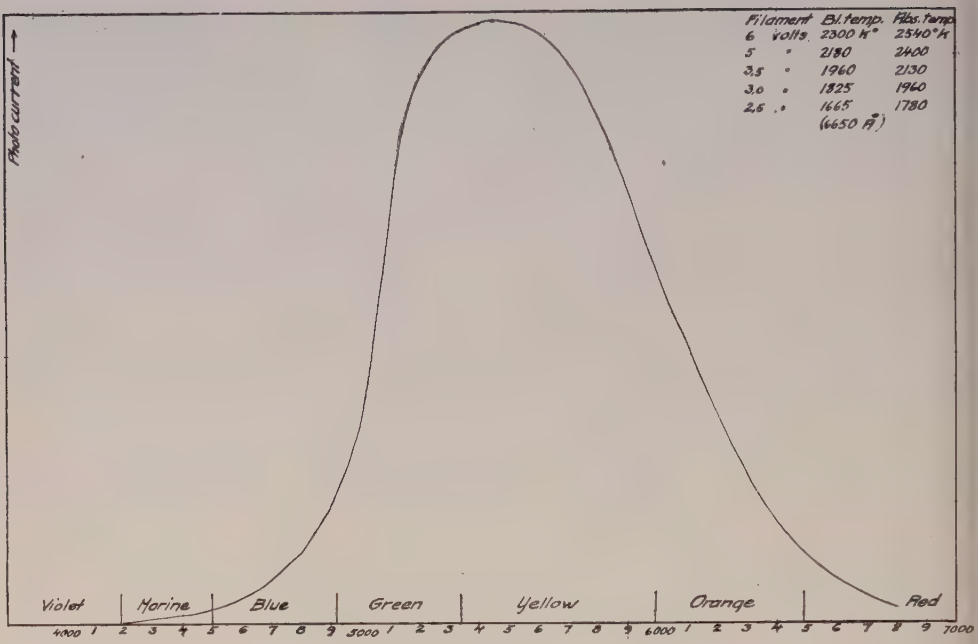


Fig. 12.

The velocity being

$$c = 299\ 792.3 \pm 1.5 \text{ km/sec.}$$

3) With practical experience of the complete geodimeter, the 1948 determinations (14) can now be corrected.

Then, in the absence of the 90-m distance, the first zero-point can be determined with tolerable accuracy from a direct measurement of the light path in the loop, as we now know. Thus from the data of the records it is evident that the first zero-point as determined at Linköping must be put forward by 7 cm.

Instead of the temperature corrections of 0.93×10^{-6} employed (Kohlrausch at 20° C), 1.1×10^{-6} is more correct.

Regarding the colour, a curve has been obtained with the actual filter and the filament voltage used as shown in Fig. 12. A value on μ_g of 1.000304 seems to be correct. The used base frequency of 8332157 Hz (younger crystal), checked a few days (12/10, tab. I) after the measurements, is correct.

The reduced distances of 1948 were

1) 1st — 1009th zero-point: 9064.377 m
2) 1st — 469th zero-point: 4208.430 m

at 5 mm humidity giving

$$1)\ c = 299\ 796.3 \pm 1.5 \text{ km/sec.}$$
$$2)\ c = 299\ 794.1 \pm 3.0 \text{ km/sec.}$$
$$\text{Mean } c = 299\ 795.8 \pm 1.5 \text{ km/sec.}$$

After reduction of the distances by the above mentioned corrections to a total of respectively 7.8 and 7.4 cm, the corrected values of 1948 are

1) 299 794.1 ± 1.3 km/sec.
2) 299 789.3 ± 2.5 km/sec.
Weighted mean: 299 793.1 ± 2.0 km/sec.

4) Finally we have the distance measurements of 1947. (18) These measurements were performed with the old geodimeter model. The crystal oscillator, however, was transferred in one piece to the new model, so that we can find the frequency used. From table I it can be seen that the frequency has increased by 150 cycles from 12th October 1948 to 14th October 1949. The running time is estimated at some 150 hours from the measuring records.

The running time during the preceding year will be about 10 hours at Lovö, 60 hours at Linköping (from the records) and 50 hours of experimental work at AGA (entries about working hours). To this must be added some 10 hours of demonstration. We get 130 hours as the total or 130 cycles. The frequency will then be 8332027 ±30 Hz.

A strong support for the correctness of this method of procedure is the fact that the frequency in 1942 as given by the manufacturer (note on the crystal) was 8.3315 Mc. 1942-1947 the crystal was employed for experimental work. A running time of 500 hours during the period mentioned seems to be reasonable.

From the paper 18), p. 108, is seen that the first zero-point in 1947 was based on direct measurement of the light path, and as we now know this is satisfactory. From the same paper, p. 110, we have the "unit" length 17.98494 m at 0° C, 773 mm Hg. If we correct for humidity and pressure and transform to vacuum value we get $\lambda = 35.98088$ m. Then we have used a refractive index of $\mu_g = 1.000302$ due to goldplated mirrors and photo cell 1 P 21 with sensitivity reaching into red. With $n = 8332027$ Hz we get the velocity obtained in 1947 to be

$$c = 299\,793.9 \pm 2.7 \text{ km/sec.}$$

The point Vårby (p. 110) being of second order, a value from the distance 7734 m should assume the weight practically equal to 0.

Assembling all the measurements in a table, we get

Year	Distance	Weight	Velocity	
1947	11025 ± 0.08 m	0.1	299 793.9 ± 2.7	93.9
1948	9064 ± 0.01	0.6	94.1 ± 1.3	93.1
1948	4208 ± 0.01	0.2	89.3 ± 2.5	
1949 May	6906 ± 0.0035	8.0	93.0 ± 0.27	93.1
1949 Sept.	6906 ± 0.0035	8.0	93.1 ± 0.28	
1949	5144 ± 0.01	1.5	94.0 ± 0.75	
1949	1762 ± 0.01	0.3	92.3 ± 1.50	

Weighted mean: 93.1 ± 0.25 km/sec

The error limits of the distances denote the geodetic accuracy.

Final result

The foregoing investigation yields the value on the velocity of light in vacuo to be

$$c = 299\,793.1 \pm 0.25 \text{ km/sec.}$$

No sign of any systematic variation with time can be discovered.

Comparison with determinations by others

MICHELSON (1926) (4, 11) obtained

$$c = 299\,798 \pm 4 \text{ km/sec.}$$

BIRGE's statistical collation in 1942 (10, 11) gave the result

$$c = 299\,776 \pm 3 \text{ km/sec.}$$

ESSEN's determination (1948) (13) by short radio waves in resonance in a short guide gave the value

$$c = 299\,792.2 \pm 4.5 \text{ km/sec.}$$

(Essen gives ±9 as the maximum error.)

ASLAKSON's value (1949) (15) by radar applied to six geodetically known distances in U. S. A. is

$$c = 299\,792.4 \pm 2 \text{ km/sec.}$$

where the inner mean error of ±1.5 km/sec., here has been increased to ±2 to allow for possible systematic errors.

Determinations of lengths

As a practical application two uncertain 1st order triangle sides in Norrland were determined.

1) The distance between the islands of Prästgrundet and Storjungfrun near Söderhamn in August:

1st night:	20203.63 m	One obs.
2nd night:	20203.57±0.04 m	Six obs.
Mean:	20203.59±0.04 m	

The coordinates give 20203.79. However the length as determined from the new Söderhamn base line via a base net and two triangles is 20203.25 m. Supposing this value to be of double the weight of that from the coordinates, the geodetically determined length would be 20203.43 m which is in tolerable agreement with the value determined with light.

2) The distance between the two fjelds Ounistunturi and Sautusvaara near Kiruna in Lappland in September

1st night	30921.67±0.17 m.	Two obs. weight 1
2nd night	.52±0.05 m.	Ten obs. weight 3
3rd night	30921.37±0.07 m.	Five obs. weight 2
Weighted mean	30921.50±0.07 m.	

The frequency was checked as usual. Provisionally adjusted coordinates gave the distance 30921.42±0.60 m.

In the Norrland measurements and with the 1762 m base line, the geodimeter was supplied with tension from a portable 400 watts benzene motor generator. For the rest the 230 volts net was used.

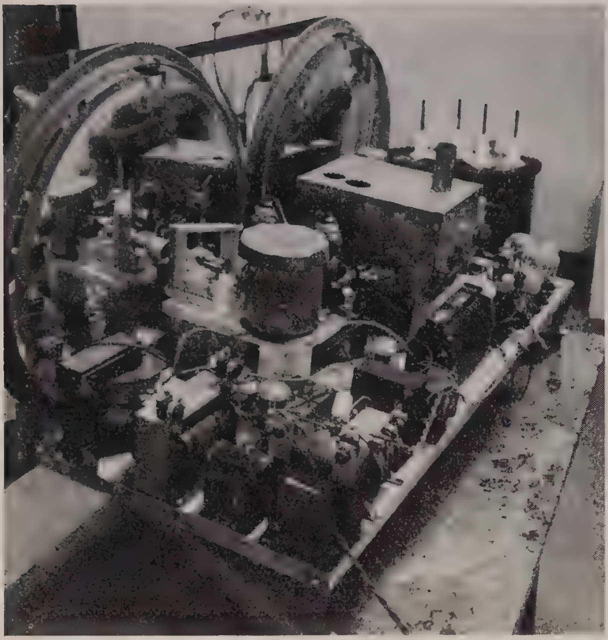
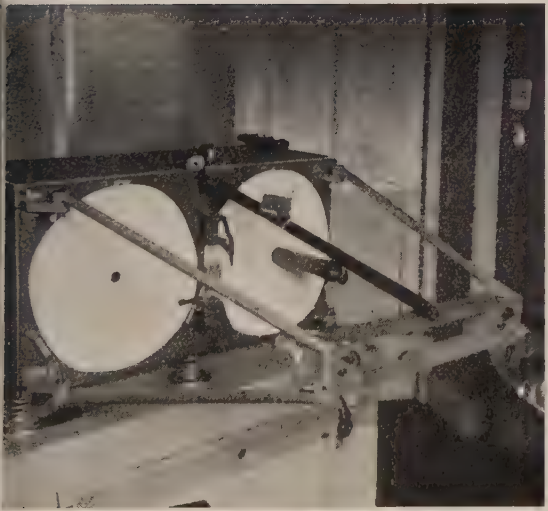


Fig. 13. The first geodimeter (1947) from behind.



4. Front of the AGA-model. Cardboard in place of h. mirrors. The var. loop on its bar in the middle. The outside optics is collapsible.

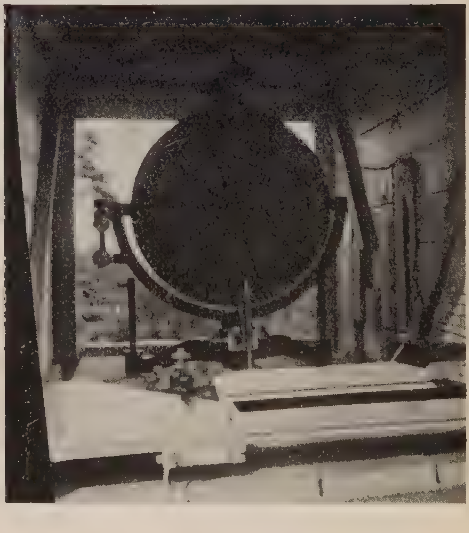


Fig. 15. Back of the plane mirror in its house. The floor is isolated on its own stand. By courtesy of Prof. Lind, Fysicum, Upsala, the mirror was placed at the disposal of the Survey.

Table I

Checking of the frequency. Cr. wh. = readings on the crystal wheel

Date in 1948–1949	Hour	Cr. wh.	Freq.
12/10	13.45	—	Start
»	15.00	0	8332 166
»	.05	5	157
»	.10	10	037
»	.15	13	8331 921
»	.20	16	779
»	.25	20	543
»	.30	23	323
»	.35	26	048
»	.40	30	8330 541
»	.45	33	005
»	.50	36	8329 171
»	.55	40	8327 258
»	16.00	43	8324 200
»	.05	46	8316 640
»	.10	50	8272 265
27/4	14.00	—	Start
»	15.35	0	8332 209
»	.40	5	2 203
»	.45	10	2 075
»	.50	15	1 860
»	.55	20	1 557
»	16.00	30	0 489
»	16.10	0	2 170
4/5	18.00	—	Start
»	19.30	0	8332 200
»	20.05	0	2 219
»	20.10	5	2 211
»	.15	7.5	2 172
»	.20	10	2 092
»	.25	12.5	1 987
»	.30	15	1 870
»	.35	17.5	1 733
»	.40	0	2 195
6/5	19.07	—	Start
»	20.00	0	8332 195
»	.05	7.5	2 152
»	20.10	12.5	8331 979
»	.15	17.5	1 740
»	.20	15	1 879
»	.25	0	2 210
»	.30	10	2 085
»	.35	5	2 210
»	.40	0	2 218
»	21.55	10	2 103
9/5	19.45	—	Start
»	20.55	10	8332 080
10/5	19.40	—	Start
»	20.55	10	8332 091
»	21.55	10	110
11/5	19.45	—	Start
»	20.55	10	8332 096
»	21.55	10	125
12/5	19.50	—	Start
»	20.55	12.5	8332 021
13/5	19.45	—	Start
»	20.55	12.5	8332 012
»	21.55	12.5	030
14/5	18.45	—	Start
»	20.15	0	8332 246
»	.20	5	2 246
»	.25	10	2 115
»	.30	15	1 890
»	.35	17.5	1 737
»	.40	12.5	1 972
»	.45	10.0	2 080
»	.50	7.5	2 176
»	.55	0.0	2 231
»	21.55	10.0	2 122
27/8	20.30	—	Start
»	21.30	10	8332 093
29/8	19.30	—	Start
»	21.30	10	2 142
10/9	19.25	—	Start
»	20.55	10	2 122
»	21.55	10	2 133
12/9	19.20	—	Start
»	22.00	17.3	8331 813
23/9	19.50	—	Start
»	20.55	10	8332 117
»	21.55	10	2 144
24/9	18.30	—	Start
»	20.55	10	8332 150
»	21.55	10	2 157
26/9	18.50	—	Start
»	20.55	10	8332 160
»	21.55	10	2 167
27/9	18.45	—	Start
»	19.55	10	8332 141
»	20.55	10	2 157
»	21.55	10	2 164
28/9	18.45	—	Start
»	19.55	10	8332 159
»	20.55	10	2 165
»	21.55	10	2 157

Table I (cont.)

Date in 1948–1949	Hour	Cr. wh.	Freq.	Date in 1948–1949	Hour	Cr. wh.	Freq.
29/9	18.40	—	Start	6/10	19.00	—	Start
»	19.55	10	8332 153	»	19.55	10	8332 141
»	20.55	10	2 168	»	20.55	10	2 166
»	21.55	10	2 173	»	21.55	10	2 175
3/10	18.40	—	Start	14/10	12.00	—	Start
»	19.55	10	8332 157	»	14.00	10	8332 211
»	20.55	10	2 163	»	14.02	0	2 339
»	21.55	10	2 176	»	14.04	10	2 203
4/10	19.00	—	Start	»	.06	5	2 327
»	19.55	10	8332 135	»	.08	10	2 207
»	20.55	10	2 140	»	.10	15	1 987
»	21.55	10	2 170	»	.12	5	2 324
5/10	18.45	—	Start	»	.14	17.5	1 847
»	19.55	10	8332 153	»	.16	10	2 184
»	20.55	10	2 164	»	.18	20	1 691
»	21.55	10	2 184	»	.20	10	2 184

Table II

90-m measurements. Readings on S_1 (in italics) and on S_2 (p. 133) in cm. Each reading in a group of four corresponds to one of the four possible position-combinations of the shifters SwI and SwII

May 15		May 19			Sept. 30			Oct. 7		
18.1	22.7	*19.6*	22.4	*17.5*	*15.3*	42.2	23.0	*15.6*	29.3	26.4
21.5	19.9	*23.7*	18.8	*21.3*	*23.0*	37.8	19.2	*21.8*	25.9	23.8
9.7	22.9	*11.2*	22.6	*10.2*	*12.3*	7.6	27.2	*17.8*	24.3	25.6
13.1	21.5	*14.8*	21.4	*14.4*	*23.4*	4.8	23.3	*24.6*	24.4	22.9
25.7	*18.1*	20.8	*19.5*	22.5	31.4	33.9	*15.6*	12.7	31.1	*14.5*
23.6	*21.2*	16.9	*21.4*	19.9	32.6	29.8	*21.0*	10.4	23.4	*19.0*
20.3	*10.3*	25.3	*9.3*	23.7	17.6	17.5	*15.1*	40.6	22.6	*18.8*
17.6	*16.2*	23.5	*14.8*	22.4	11.4	12.9	*20.0*	36.6	20.5	*24.2*
20.7	21.5	*19.7*	20.9	*18.1*	24.0	*14.9*		23.7	*16.5*	25.4
21.0	18.4	*23.0*	19.4	*22.5*	21.0	*19.1*		20.6	*19.8*	23.0
8.9	22.0	*9.8*	21.9	*9.8*	28.0	*13.4*		29.8	*17.7*	27.7
10.9	20.9	*14.2*	21.0	*13.3*	26.0	*22.8*		26.8	*23.5*	24.8
22.9	*16.8*	22.1	*17.7*		*12.3*	9.8		*14.9*	30.2	25.8
19.4	*20.8*	17.0	*22.8*		*21.3*	3.0		*18.6*	24.3	20.8
23.9	*10.7*	25.3	*10.8*		*15.2*	47.8		*18.3*	24.5	25.8
23.6	*13.4*	22.5	*14.8*		*23.0*	42.2		*25.0*	20.1	24.6
17.7		*21.3*	22.3		27.5	30.2		21.7	19.3	*15.9*
19.3		*23.8*	19.8		21.7	25.4		18.5	14.2	*21.7*
10.7		*10.8*	23.4		26.7	22.1		31.3	30.8	*14.7*
11.6		*14.3*	21.3		24.0	17.4		26.5	28.9	*19.9*
21.5		21.0	*20.7*		6.4	*11.6*		28.4	*17.7*	22.3
20.1		18.1	*22.1*		1.0	*19.8*		25.2	*20.5*	21.9
22.6		26.0	*9.7*		46.8	*15.9*		25.2	*14.5*	25.8
20.5		23.2	*14.9*		42.6	*21.9*		22.6	*18.2*	24.9
17.1		*21.3*	21.9		*15.3*	19.6		*14.9*	20.8	21.9
21.7		*21.9*	19.8		*19.9*	17.0		*19.9*	17.7	20.7
10.9		*9.9*	22.8		*15.7*	28.6		*16.7*	31.6	27.5
15.9		*14.9*	21.4		*19.3*	25.2		*24.0*	27.4	24.2
										12.5
										17.7
										15.6
										22.4

Table III

7 km measurements. Readings on S_1 (p. 133) in italics and on the crystal wheel. Each reading in a group of four corresponds to one of the four possible position-combinations of the shifters SwI and SwII. The first figures each day are date and hour of start. The last figures are hours of conclusion

6/5	*12.5*	*10.1*	*9.4*	*20.3*	9.8	12.7	6.9	10.3
21.40	*15.3*	*14.6*	*18.7*	*21.7*	*14.4*	9.3	18.4	4.8
	18.9		*19.8*	*10.3*	*11.0*	*19.5*	11.4	13.2
11.6	*21.4*	23.20	12.3	*14.6*	*19.6*	*22.5*	*19.3*	10.9
13.3	10.7		13.8	15.4	*17.7*	*9.8*	*24.3*	*20.0*
11.2	7.0		8.8	12.6	8.9	*13.5*	*13.2*	*24.4*
7.9	15.7	10/5	11.4	13.6	11.8	13.5	*17.1*	*6.3*
16.4	13.4		*15.3*	11.3	8.3	10.4		*14.8*
12.6	*12.9*	20.45	*11.4*	*19.7*	11.1	13.6	20.40	10.7
20.2	*16.0*		*20.2*	*21.2*	*13.7*	9.7		9.0
16.9	*19.3*	*16.8*	*16.7*	*11.5*	*11.6*	*18.9*		13.2
12.3	*21.4*	*13.0*	12.7	*14.7*	*23.1*	*20.4*	26/9	11.3
8.7	10.1	*22.2*	14.0	14.8	*20.8*	*8.8*		13.4
11.7	7.3	*17.4*	9.8	12.5	9.5	*11.0*	19.40	11.2
8.7	15.5	12.2	11.3	14.1	12.5	13.8		9.9
21.0	13.6	13.6	*12.0*	11.6	8.8	10.5	*25.6*	8.0
22.1	*14.2*	7.7	*10.7*	*20.0*	11.3	14.0	*27.7*	*18.5*
11.3	*14.1*	11.4	*22.0*	*23.1*	*14.3*	8.9	*14.0*	*24.4*
13.3	*17.8*	*14.9*	*17.6*	*10.5*	*11.4*	*19.8*	*16.6*	*10.3*
11.7	*20.7*	*12.0*		*14.6*	*21.7*	*24.6*	14.2	*15.3*
9.3	10.8	*21.6*	23.00		*19.6*	*8.3*	11.4	14.8
12.2	6.6	*20.1*		23.00		*13.0*	8.9	12.6
8.6	15.7	10.9			23.00	13.6	4.0	9.1
17.7	13.6	14.1	11/5			9.6	9.5	7.5
19.6	*12.8*	10.0		13/5		13.5	5.0	6.4
12.2	*14.3*	12.2	21.50		14/5	9.3	13.4	10.6
14.3	*17.6*	*14.8*		21.30		*19.6*	11.1	11.9
11.7	*20.8*	*12.1*	*19.4*		21.35	*24.4*	*22.7*	13.9
7.6	11.7	*19.4*	*24.6*	*13.7*		*9.8*	*25.8*	*18.2*
12.7	5.6	*18.0*	*9.5*	*11.1*	*20.2*	*14.1*	*12.4*	*25.0*
8.8	16.5	11.3	*16.8*	*21.5*	*23.4*		*15.9*	*8.7*
20.4	12.8	12.9	15.5	*17.5*	*9.2*	23.30	9.5	*18.2*
23.2	*11.7*	9.9	12.1	10.0	*14.8*		6.6	
9.7	*14.7*	11.8	13.8	12.5	12.1		13.6	22.30
15.5	*21.0*	*14.2*	10.7	8.0	6.7	24/9	10.7	
	21.4	*11.4*	*17.6*	11.9	14.3		9.7	
23.00		*21.8*	*22.1*	*13.7*	10.3	19.25	6.2	27/9
	23.00	*20.2*	*12.7*	*10.5*	*17.2*		13.3	
		11.1	*17.4*	*21.5*	*22.4*	*21.3*	11.0	19.50
8/5		13.5	15.6	*19.2*	*9.2*	*28.6*	*22.1*	
		9.8	12.0	10.4	*13.8*	*12.9*	*27.2*	*15.7*
21.45	9/5	12.5	13.9	12.1	12.1	*18.4*	*13.3*	*21.5*
		13.2	11.3	9.0	8.4	11.5	*17.5*	*11.1*
14.7	22.55	*11.0*	*18.0*	11.3	12.1	5.3	*18.2*	*21.4*
13.0		*21.8*	*22.8*	*12.2*	8.0	16.0	*25.3*	10.0
11.8	13.0	*17.2*	*12.2*	*9.6*	*19.1*	11.8	*10.4*	4.0
9.1	12.1	14.5	*16.7*	*21.0*	*21.0*	*20.3*	*14.0*	15.7
10.7	13.8	17.2	14.9	*17.5*	*10.8*	*22.8*	10.7	12.1
8.1	12.6	−6.5	13.5	10.3	*13.5*	*14.7*	7.0	8.6
15.6	*18.2*	8.3	13.5	13.2	13.3	*17.8*	12.9	4.0
13.2	*19.3*	*13.7*	11.0	8.0	8.5	12.3	10.2	16.9

Tabelle III (cont.)

13.4	16.3	10.7	16.4	10.3	13.8	8.6	14.4	*22.3*
11.4	14.4	13.7	15.7	*13.4*	8.9	13.4	4.4	15.3
16.9	*12.8*	8.1	8.3	*19.8*	*14.6*	11.2	13.5	10.6
20.3	*19.5*	14.7	(−5.8)	*17.6*	*18.0*	11.2	3.9	12.2
26.4	*16.8*	11.3		*23.0*	*17.1*	8.8	12.5	4.2
9.3	*25.2*	13.5	20.20	13.4	*23.4*	13.4	−5.8	15.5
−5.8	14.2	9.8		10.5	13.1	10.7	15.1	11.3
15.9	9.4	*17.7*		12.6	9.4	*12.3*	10.3	11.3
13.1	13.7	*21.2*	29/9	9.7	13.0	*17.1*	11.1	7.1
7.2	8.5	*8.7*		13.2	9.4	*18.0*	5.8	14.4
−5.8	*11.0*	*17.2*	19.50	10.5	13.2	*25.2*	14.6	10.8
17.3	*22.2*			12.3	9.7		10.4	12.4
14.7	*9.6*	22.20	*27.2*	9.2	14.3	22.20	10.5	7.1
11.5	*20.0*		*29.7*	*10.9*	10.5		4.0	14.9
19.7	13.9		*4.6*	*15.7*	*10.4*		15.8	9.7
14.2	10.5	28/9	*10.0*	*18.7*	*15.8*	3/10	12.3	12.7
20.4	13.6		13.0	*23.4*	*21.2*		10.8	8.4
7.4	10.2	19.50	8.5	13.4	*26.5*	20.20	4.0	*11.4*
−5.8	14.1		15.5	10.5	12.7		15.4	*16.5*
17.7	10.0	15.7	10.0	12.5	8.8	*10.9*	10.9	*16.0*
15.3	12.9	9.3	12.4	9.6	13.7	*14.4*	*11.1*	*22.0*
9.7	8.4	8.3	9.8	13.7	11.3	*17.8*	*18.6*	
−5.8	14.5	4.0	12.4	8.7	12.0	*23.4*	*16.3*	22.30

Table IV

Atmospheric conditions

1949		Temperature		Pressure[1] mm Hg	Temp. of barom
Date	Hour	Geod.	Mirror		
6/5	21.02	+9°.2 C	+8°.6 C	749.3	+10°.8 C
»	21.35	7.9	8.6		
»	22.20	7.7	7.6		
»	23.05	7.6	7.4	749.7	9.5
8/5	21.25	1.6	2.6	754.9	4.1
»	22.15	1.2	1.9		
»	23.00	0.4	1.6	755.0	3.3
9/5	21.00	3.9		761.4	5.6
»	22.50		2.2		
»	23.15		1.6		
10/5	20.57	9.1	8.9	764.6	10.1
»	21.45	9.0	9.1		
»	22.20	8.6	8.6		
»	22.50	7.7	7.8	765.0	9.6
11/5	21.40	8.2	7.8	768.8	9.5
»	22.05	7.8	7.8		
»	22.40	7.6	6.8		
»	23.10	7.1	6.8	768.8	8.7
13/5	21.20	11.4	11.4	764.6	14.1
»	22.00	11.4	11.7		
»	22.35	11.0	10.3		
»	22.55	10.8	10.6	764.6	13.3
14/5	21.25	9.8	9.7	762.3	12.8
»	21.50	8.9	8.3		
»	22.30	8.7	8.1		
»	23.20	8.3	7.5	762.1	10.6
24/9	19.08		12.6		
»	19.20	11.1		767.7	12.9
»	20.10	10.6	12.1	767.9	12.7
»	20.55	11.9	11.9	767.8	12.9
»	22.00	11.2	10.9	768.0	12.8
26/9	19.20	15.6	15.6	764.2	15.9
»	19.55	15.2	15.3	764.2	15.9
»	20.50	14.8	14.7	764.1	16.2
»	22.00	14.2	14.7	764.1	?
»	22.15	14.0			
»	22.30		14.7		
27/9	19.20	13.3	13.7	763.4	14.6
»	20.15	12.2	12.5		
»	21.00	11.7	12.0	763.1	12.7
»	22.00	11.7	11.9	763.0	12.2
»	22.00	11.2	11.9		

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[1] The pressure figures are unreduced readings. Lat = 59°32′, 26 m. a. s. Brass scale, correct at 0° C.

Table IV (cont.)

1949		Temperature		Pressure[1] mm Hg	Temp. of barom
Date	Hour	Geod.	Mirror		
28/9	19.05	+10°8 C		764.0	+12°6 C
»	20.00	10.4	+11°8 C	764.1	12.2
»	20.35		10.1		
29/9	19.35	13.8	13.0	758.9	14.6
»	20.00	13.6	12.9	758.6	14.5
»	21.00	14.2	14.2	758.3	14.7
»	22.00	14.1	13.9	757.8	14.6
»	22.25	14.3	14.3	757.6	15.1
3/10	19.45	9.9	9.9	747.3	10.6
»	21.00	8.9	9.1	749.3	9.8
»	22.30	8.1	7.8	751.7	9.5

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[1] The pressure figures are unreduced readings. Lat = 59°32′, 26 m. a. s. Brass scale, correct at 0° C.

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Tryckt den 21 april 1950

Uppsala 1950. Almqvist & Wiksells Boktryckeri AB

Emendations

P. 129: 3rd line: .001–.001. Read: .01–.01
P. 129: The phototube is an 1 P 21 (R. C. A.)
P. 133: 2nd line fr. f.: (p. 028). Read: (p. 146)
P. 136: 3rd line bel. the tables: fr. p. 9, 23). Read: fr. p. 127, 141)
P. 136: 10 lines further below: (p. 31). Read: (p. 149)
P. 141: column of weight: 0.3. Read: 0.4
P. 141: In the middle of the page: 1 P 21. Read: 1 P 22
P. 147: 3rd column, near the bottom: —6.5 put in Fig. 9 is to be read off: —2.3 cm